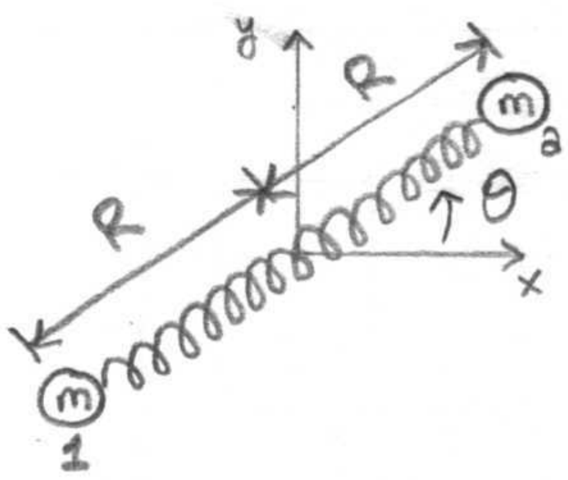
11.6

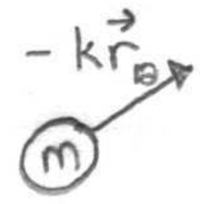


a) Yes, linear momentum is conserved.
$\vec{F}_{12} = F(r_{12}) \frac{\vec{r}_{12}}{r_{12}}$, $\vec{F}_{12} = -\vec{F}_{21}$, so $\Sigma \vec{F} = 0$

b) We know $\Sigma \vec{F}_G = m_T \vec{a}_G = -k\vec{r}_{12} - k\vec{r}_{21}$, and since $\vec{r}_{12} = -\vec{r}_{21}$, $k\vec{r}_{21} - k\vec{r}_{21} = m_T \vec{a}_G = \vec{0}$

$\therefore \; \vec{a}_G = 0$

c) Mass 1:



Mass 2:

d) $m_1 \vec{a}_1 = -k\vec{r}_{12} = k(\vec{r}_2 - \vec{r}_1)$, where $\vec{r}_i = x_i \hat{\imath} + y_i \hat{\jmath}$

$$\therefore \quad \ddot{x}_1 = \frac{k}{m}(x_2 - x_1)$$
$$\ddot{y}_1 = \frac{k}{m}(y_2 - y_1)$$

$$m_2 \vec{a}_2 = -k\vec{r}_{21} = k(\vec{r}_1 - \vec{r}_2) \quad \therefore \quad \ddot{x}_2 = \frac{k}{m}(x_1 - x_2)$$
$$\ddot{y}_2 = \frac{k}{m}(y_1 - y_2)$$

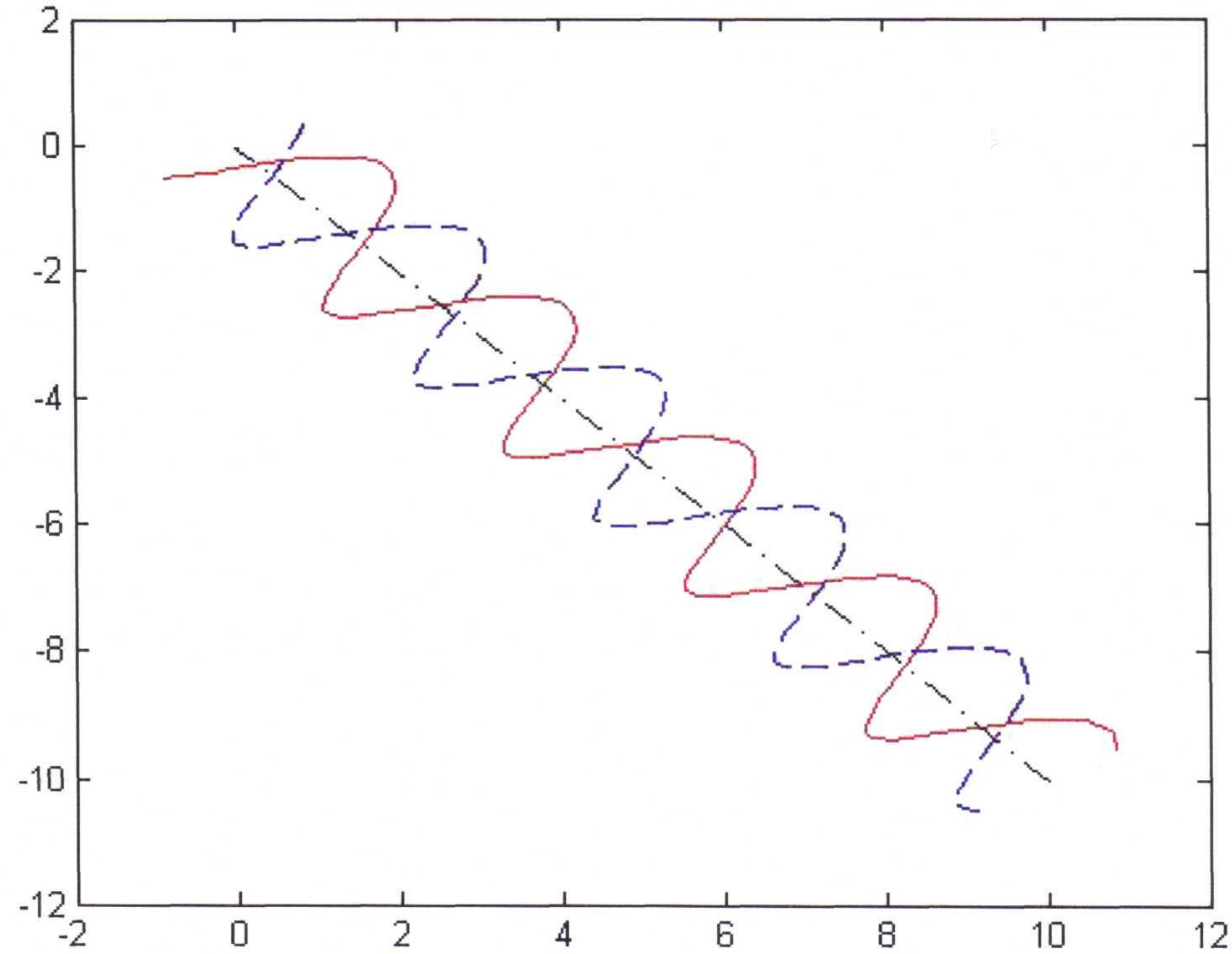
e)
$$E_k = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2)$$
$$= \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2)$$
$$E_p = \frac{1}{2}k\left[(x_2 - x_1)^2 + (y_2 - y_1)^2\right]$$

$$E_k + E_p = \text{constant} = E_{p0} = \frac{1}{2}k(2R)^2 = 2kR^2$$

f) See attached Matlab code and plot, assuming initial conditions shown and

$$\vec{v}_{10} = \hat{\imath} \;, \quad \vec{v}_{20} = -\hat{\jmath}$$

```
function Prob116()
% Problem 11.6 Solution
% April 1, 2008

% VARIABLES
R= 1;
th= 30; % theta in degrees
k= 1; % spring stiffness
m= 1; % masses

% Initial Conditions
r01= -R*[cosd(th) sind(th)]';
r02= R*[cosd(th) sind(th)]';
v01= [1 0]';
v02= [0 -1]';

z0= [r01; r02; v01; v02]; % pack variables

tspan= [0 20];

[t zarray]= ode45(@rhs,tspan,z0,[],k,m);

% Unpack variables
r1= zarray(:,1:2);
r2= zarray(:,3:4);
rG= (r1+r2)/2;

plot(r1(:,1), r1(:,2), 'r');
hold on;
plot(r2(:,1), r2(:,2),'b--');
plot(rG(:,1), rG(:,2),'k-.');

end

% THE DIFFERENTIAL EQUATIONS (RIGHT HAND SIDE)
function zdot = rhs(t,z,k,m)

% Unpack variables
r1= z(1:2);
r2= z(3:4);
v1= z(5:6);
v2= z(7:8);

% The equations
r1dot= v1;
v1dot= k/m*(r2-r1);
r2dot= v2;
v2dot= k/m*(r1-r2);

% Pack the rate of change variables
zdot= [r1dot; r2dot; v1dot; v2dot];

end
```



Problem 11.6 Plot

11.10

a) See attached Matlab code and plots for (a) - (d), recognizing that:

$$m\ddot{a}_1 = \frac{Gm^2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1) + \frac{Gm^2}{|\vec{r}_3-\vec{r}_1|^3}(\vec{r}_3-\vec{r}_1)$$

$$\therefore \quad \ddot{\vec{a}}_1 = Gm\left(\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3} + \frac{\vec{r}_3-\vec{r}_1}{|\vec{r}_3-\vec{r}_1|^3}\right)$$

We can turn this, and perform similar linear momentum balance on (2) and (3), into six first-order vector differential equations:

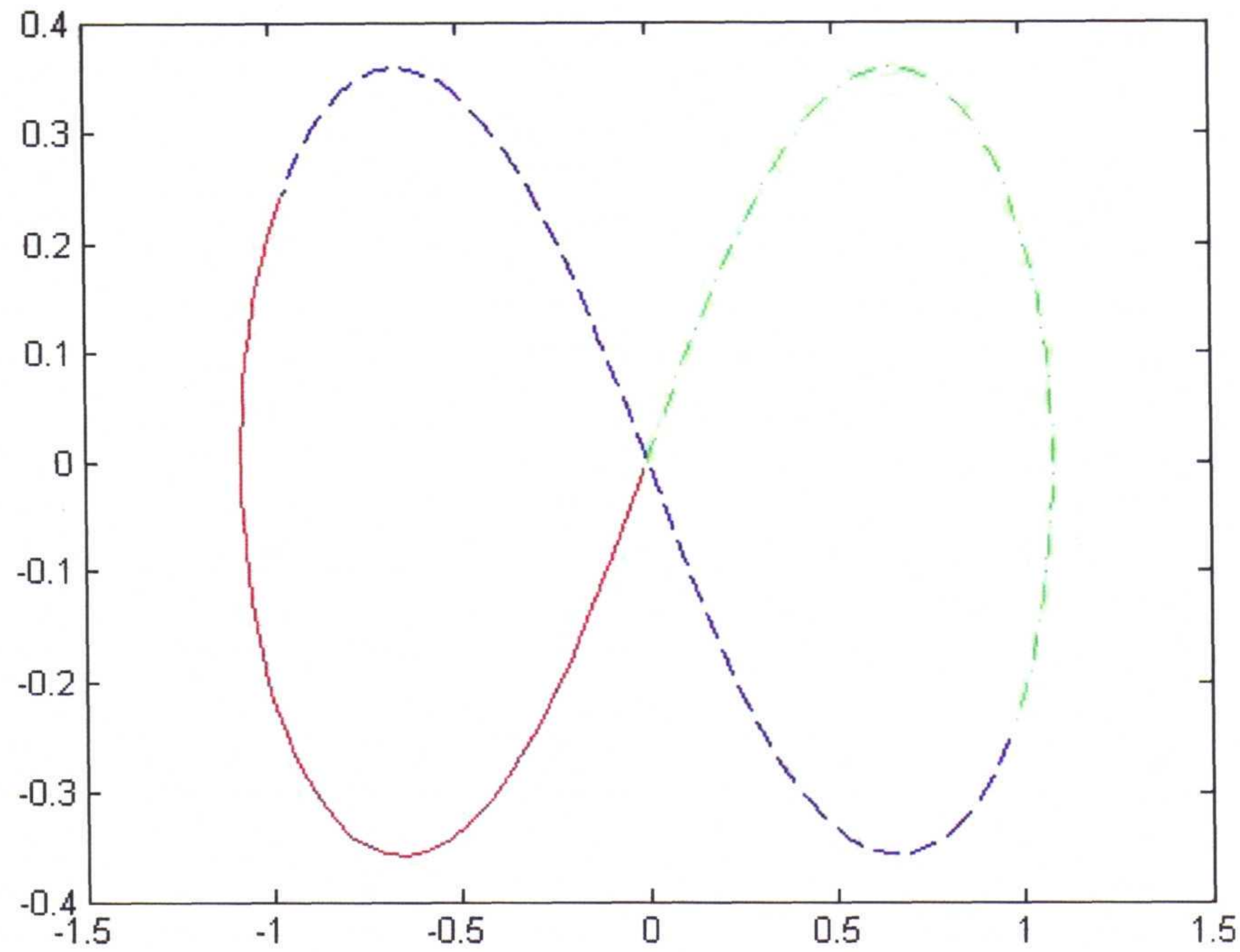
$$\dot{\vec{r}}_1 = \vec{V}_1 \qquad \dot{\vec{V}}_1 = Gm\left(\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3} + \frac{\vec{r}_3-\vec{r}_1}{|\vec{r}_3-\vec{r}_1|^3}\right)$$

$$\dot{\vec{r}}_2 = \vec{V}_2 \qquad \dot{\vec{V}}_2 = Gm\left(\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|^3} + \frac{\vec{r}_3-\vec{r}_2}{|\vec{r}_3-\vec{r}_2|^3}\right)$$

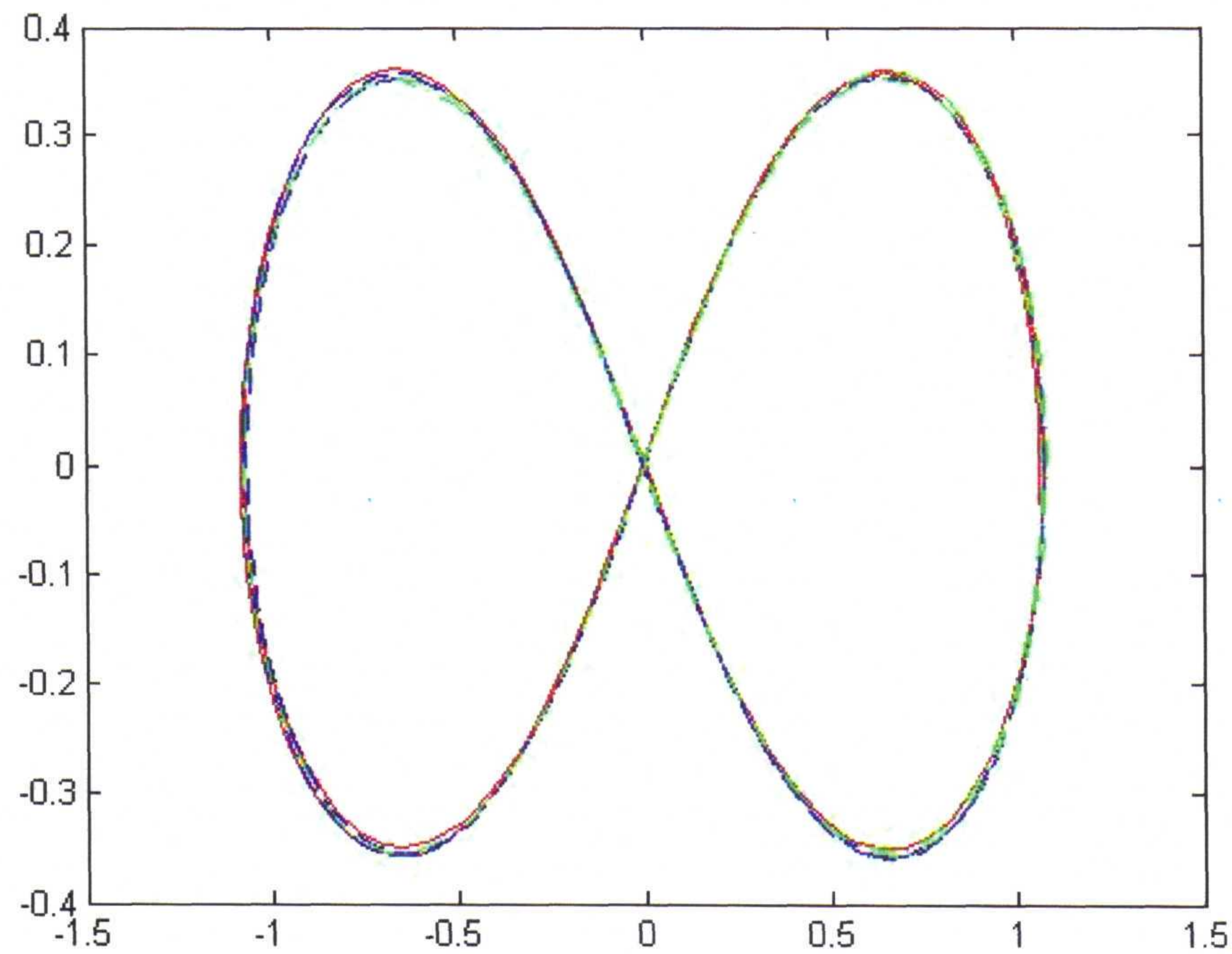
$$\dot{\vec{r}}_3 = \vec{V}_3 \qquad \dot{\vec{V}}_3 = Gm\left(\frac{\vec{r}_1-\vec{r}_3}{|\vec{r}_1-\vec{r}_3|^3} + \frac{\vec{r}_2-\vec{r}_3}{|\vec{r}_2-\vec{r}_3|^3}\right)$$

From plots on subsequent pages, we can see that these initial conditions provide for a very specific displacement function for each mass. If these conditions are modified slightly, as in (c) and (d), the displacement plot is very different.

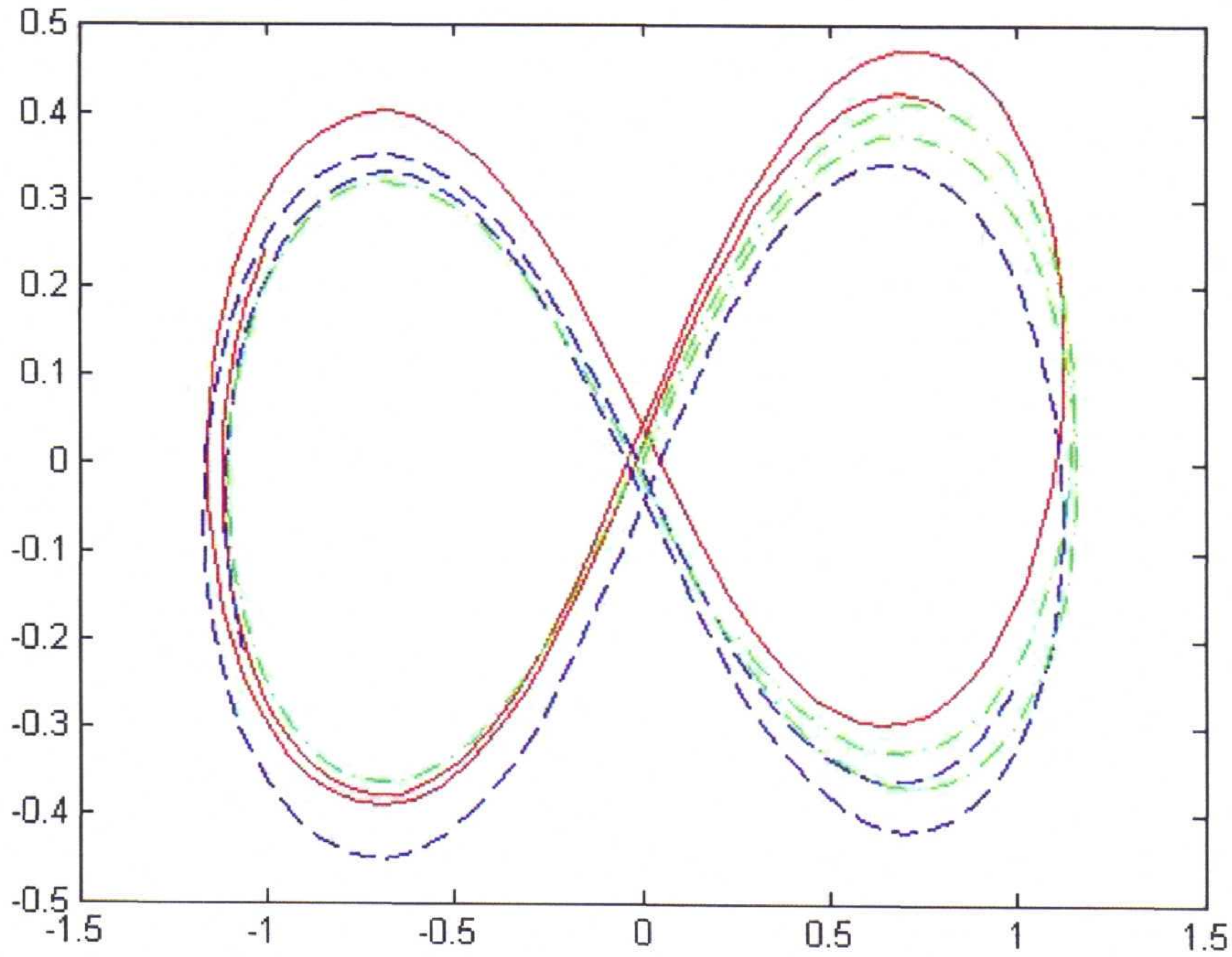
```
function Prob1110()
% Problem 11.10 Solution
% April 1, 2008

% VARIABLES
G= 1;
m= 1;

% Initial Conditions
r01= [-0.97000436 0.24308753]'; r02= -r01; r03= [0 0]';
v03= [0.93240737 0.86473146]'; v01= -1/2*v03; v02= -1/2*v03;

z0= [r01; r02; r03; v01; v02; v03]; % pack variables

tspan= [0 10];

[t zarray]= ode45(@rhs,tspan,z0,[],G,m);

% Unpack variables
r1= zarray(:,1:2);
r2= zarray(:,3:4);
r3= zarray(:,5:6);

plot(r1(:,1), r1(:,2), 'r');
hold on;
plot(r2(:,1), r2(:,2),'b--');
plot(r3(:,1), r3(:,2),'g-.');

end

% THE DIFFERENTIAL EQUATIONS (RIGHT HAND SIDE)
function zdot = rhs(t,z,G,m)

% Unpack variables
r1= z(1:2);
r2= z(3:4);
r3= z(5:6);
v1= z(7:8);
v2= z(9:10);
v3= z(11:12);

% The equations
r1dot= v1; r2dot= v2; r3dot= v3;
v1dot= G*m*((r3-r1)/(sqrt(sum((r3-r1).^2)))^3+...
    (r2-r1)/(sqrt(sum((r2-r1).^2)))^3);
v2dot= G*m*((r1-r2)/(sqrt(sum((r1-r2).^2)))^3+...
    (r3-r2)/(sqrt(sum((r3-r2).^2)))^3);
v3dot= G*m*((r1-r3)/(sqrt(sum((r1-r3).^2)))^3+...
    (r2-r3)/(sqrt(sum((r2-r3).^2)))^3);

% Pack the rate of change variables
zdot= [r1dot; r2dot; r3dot; v1dot; v2dot; v3dot];

end
```


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

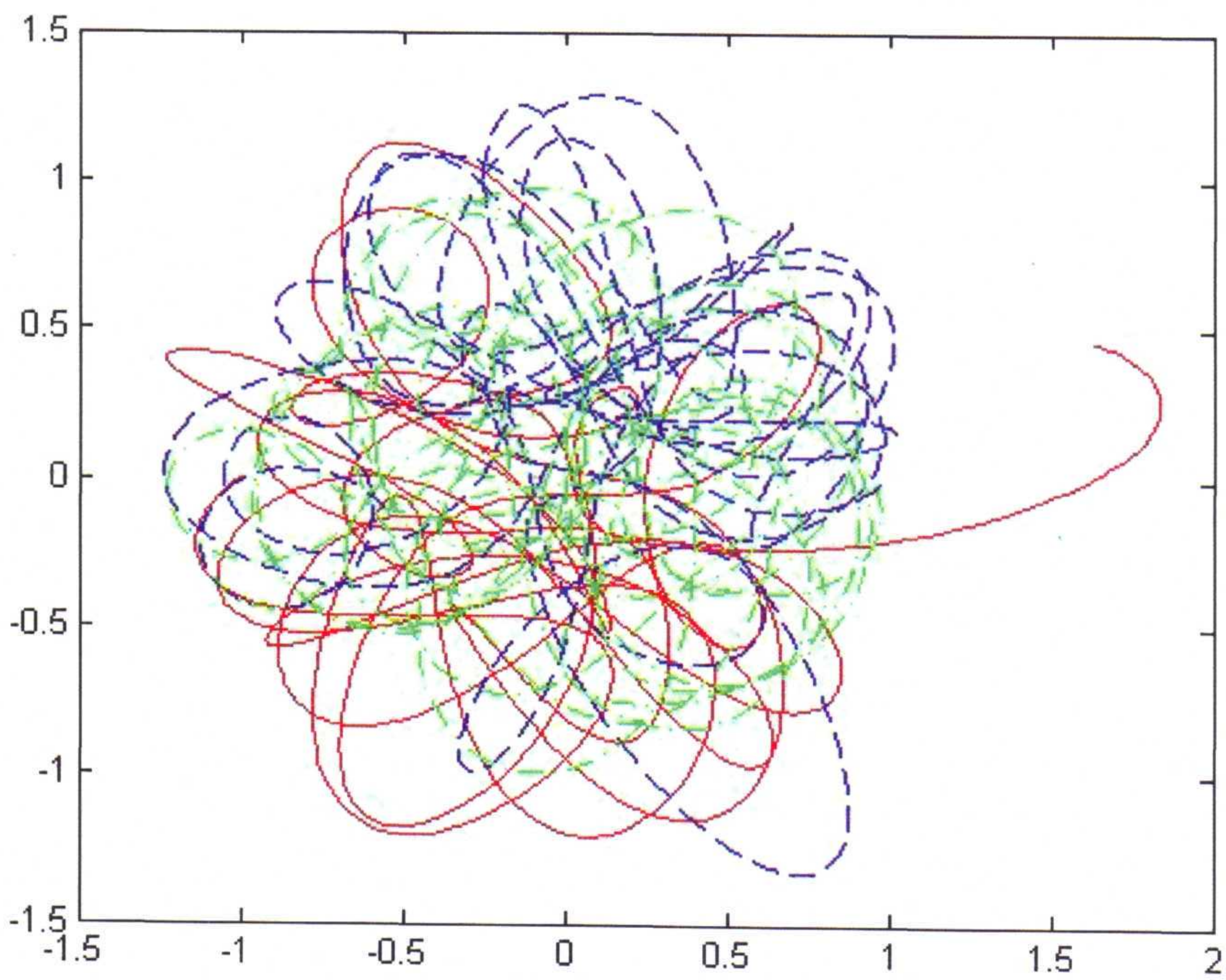
(a)


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

(b)


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

(c)

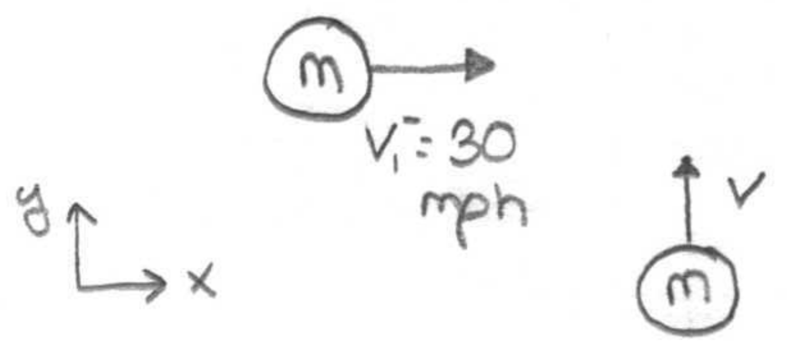

1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2

(d)

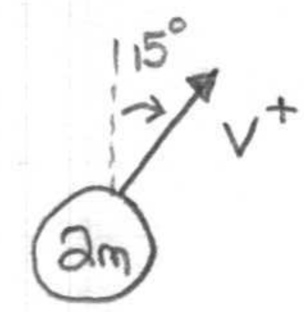
```
% Two-Particle Collisions
% Problem 11.20 Solution
% April 1, 2008

theta = 45;           % angle (degrees) between n and plus x axis
nx = cosd(theta);
ny = sind(theta);
n = [nx ny]';         % impulse direction
v1bef = [10 20]';     % vel of m1 before collision
v2bef = [-5  3]';     % vel of m2 before collision
m1 = 3;  m2 = 19;     % values of two masses
e = .5;               % coefficient of restitution

% Write governing equations in form of Az=b
% where z is a list of unknowns representing
% the particle velocities after the collision
% and the magnitude of the impulse.

A = [ m1    0   m2  0    0         % x comp of lin mom bal
      0    m1   0   m2   0         % y comp of lin mom bal
     -nx  -ny   nx  ny   0         % restitution equation
      0     0   m2  0   -nx        % impulse-momentum for m2, x comp
      0     0   0   m2  -ny];      % impulse-momentum for m2, y comp

b = [m1*v1bef + m2*v2bef;          % x & y comps of lin mom bal for syst
     -e*sum((v2bef-v1bef).*n);     % restitution equation
     m2*v2bef];                    % impulse-momentum for m2, x & y comps

z= A\b;

% Type out the solution (crudely).
disp('   v1xaft     v1yaft    v2xaft    v2yaft       P');
disp(z');

%   ANSWER:
%   v1xaft      v1yaft    v2xaft    v2yaft      P
%  -10.7273   -0.7273   -1.7273    6.2727   87.9384
```

11.22

Before collision:



After collision:



$\vec{v}_1^{\,-} = 30\,\hat{i}$ $\qquad$ $\vec{v}_1^{\,+} = \vec{v}_2^{\,+} = v^+(\sin 15°\,\hat{i} + \cos 15°\,\hat{j})$

$\vec{v}_2^{\,-} = v\,\hat{j}$

LMB: (x) $\quad 2\cancel{m}v^+\sin 15° = \cancel{m}(30)$

$$v^+ = \frac{30}{2\sin 15°} = 57.96 \text{ mph}$$

(y) $\quad 2\cancel{m}v^+\cos 15° = \cancel{m}v$

$$\therefore\ v = 2v^+\cos 15° = \boxed{112 \text{ mph}}$$